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TESTING SCALING AND QCD WITH MUON BEAMS*

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ABSTRACT

Scaling in muon-nucleon scattering suggests that nucleons are made of pointlike, non-interacting, light quarks. This scaling will go away at higher energies if the quarks are characterized by some non-zero size, a. Specifically, scaling for $Q^2 \lesssim (3a)^{-2}$ will disappear completely for $Q^2 \gtrsim a^{-2}$. Alternatively, scaling may persist, at least to the accuracy that it is currently observed. Then we may ask if the observed violations are indeed those predicted by quark-gluon gauge theory (QCD). However, there are other, more dramatic predictions of QCD which will be easier to test. The most striking of these is the prediction of large transverse momenta of order $Q^2/\log 4Q^2$ (transverse to the nominal jet axes.)

There are some interesting possibilities for weak scattering of muons. However, the interesting multiple μ production will come from weak decays of associatedly produced heavy quarks in electromagnetic scattering. QCD gives a specific prediction for the dominant contribution to associated production of new quantum numbers.

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1. Scaling Goes Away

Scaling in $\mu + p \to \mu + X$ is the approximate Q^2-independence of the electromagnetic structure functions.

$$d\sigma = \ldots F(x, Q^2) \approx \ldots F(x).$$

This is universally interpreted as indicating that nucleons are made of pointlike (non-interacting) light quarks, at least approximately. (While many different languages are used for describing this phenomenon, I believe that virtually all of them are physically equivalent).

Will this phenomenon persist, or is there some further structure to be uncovered at higher Q^2 (shorter distance)? To give an indication of what that would look like experimentally, I would like to review the parton description of scaling and then incorporate the possibility that quarks are themselves composite.

In the parton-impulse approximation[1] we write the lepton-hadron differential cross section $d\sigma_{\text{hadron}}$ as

$$d\sigma_{\text{hadron}}(x,Q^2) = \int dz\ f(z)\ d\sigma_{\text{quark}}\left(\frac{x}{z}, Q^2\right),$$

ie. an incoherent sum over partons of momentum fraction z. $(x/z = Q^2/2z\ p \cdot q)$. See fig. 1.

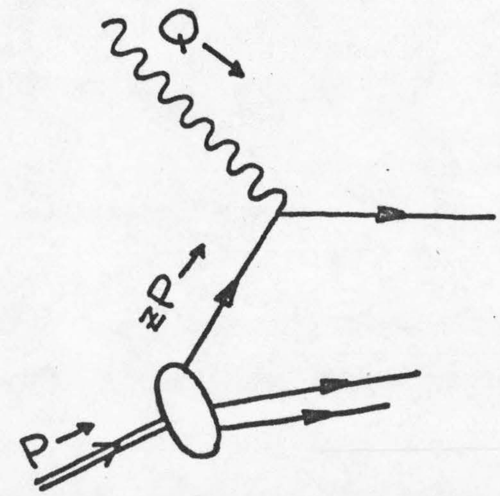


fig. 1

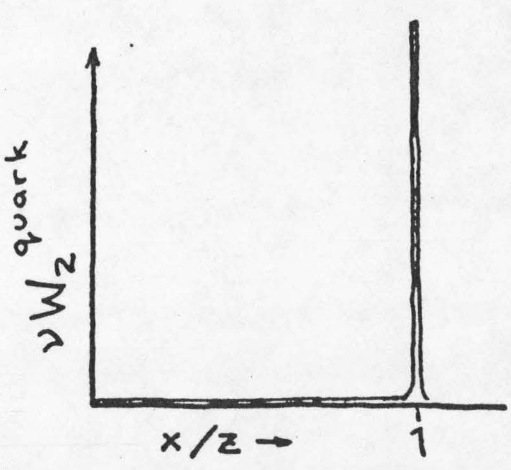


fig. 2

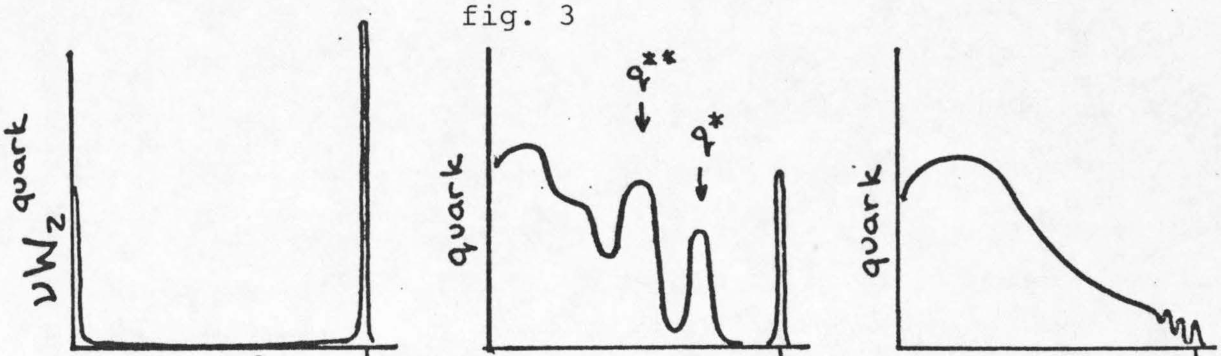
Ignoring quark interactions, we put

$$d\sigma_{\text{quark}} \propto \delta(\frac{x}{z} - 1),$$

independent of Q^2. That is to say that we only consider elastic quark scattering. See fig. 2.

What if quarks are composite, of a size $a = \lambda^{-1}$, where λ is a mass? For $Q^2 \leq \lambda^2/10$, the virtual photon will still only see the quark as a point, as in fig. 3.

fig. 3



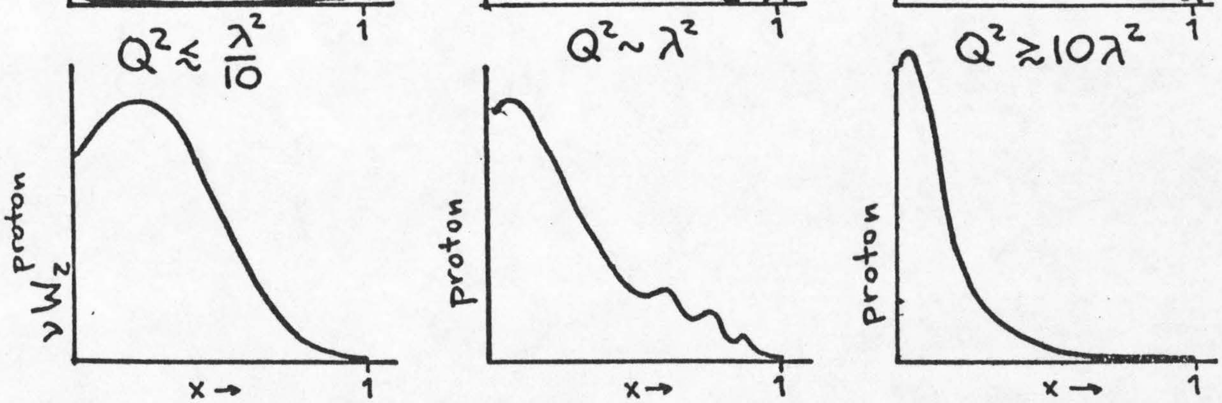


fig. 4

But for $Q^2 \sim \lambda^2$, a form factor will depress elastic scattering. Quasi-elastic and inelastic scattering are possible. For $Q^2 \geq 10\lambda^2$, scaling will set in again, but this time dominated by inelastic quark scattering (or elastic scattering off quark constituents).

Fig. 4 shows the convolution of the quark structure function with the quark distribution function to give the hadron structure function. Note that there will be new baryon resonances of the form q^*qq, where q^* is an excited quark.

As yet there is no experimental sign of such structures for $Q^2 \lesssim 50$ GeV^2. But the picture may look totally different by $Q^2 \sim 500$ GeV^2. A similar limit comes from e^+e^- annihilation, where no substructure is seen for $s \lesssim 50$ GeV^2.

Let me digress to note what quark substructure would look like in e^+e^-. The scaling ratio R would be given by

$$R \approx \sum_{\text{quarks}} Q_i^2 \quad \text{for } s \leq \lambda^2/10.$$

Even though the electromagnetic current is composed of sub-quarks, for $s \leq \lambda^2/10$ coherence over the whole quark will give the quark charges. For $s \geq 10\lambda^2$, the sub-quarks will be produced incoherently, giving

$$R \approx \sum_{\substack{\text{sub-}\\\text{quarks}}} Q_i^2 \quad \text{for } s \geq 10\lambda^2 .$$

This number may be larger or smaller than the analogous quark sum. Of course, it would be most striking if it were smaller. See Fig. 5.

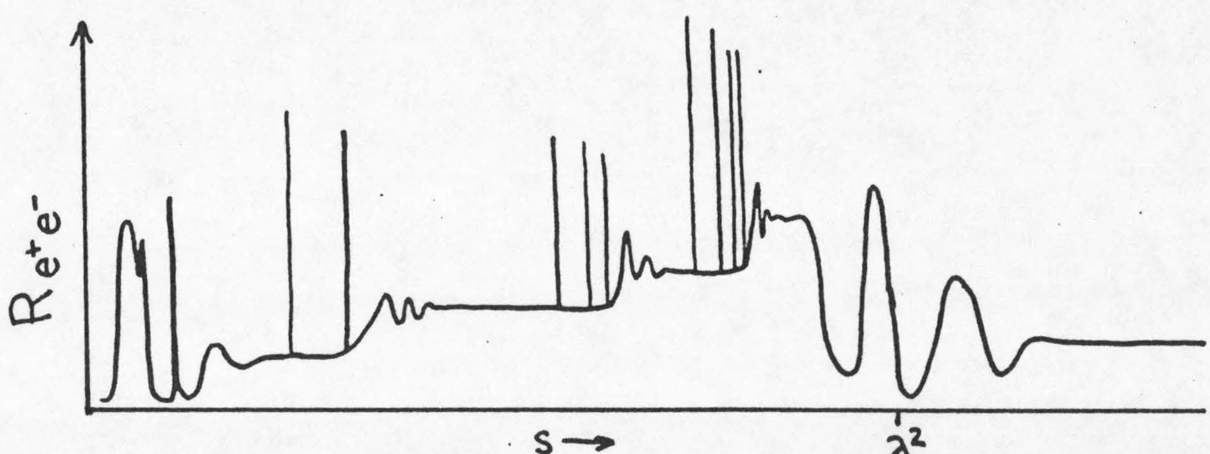


fig. 5

The real world may be yet more complicated. Muons may be composite on the scale of λ^2, or even sooner!

2. Scaling Persists

QCD is a theory which is consistent with quantum mechanics, relativity, and approximate scaling. We know how to extract definite predictions, and these have been so far verified. As it is unique in fitting this description, it is the only theory I will here consider.

What are these predictions? What do they look like at 1000 GeV? How can we test the theory?

In QCD, there are gluon radiative corrections (bremsstrahlung) to $d\sigma_{\text{quark}}$, indicated in Fig. 6.

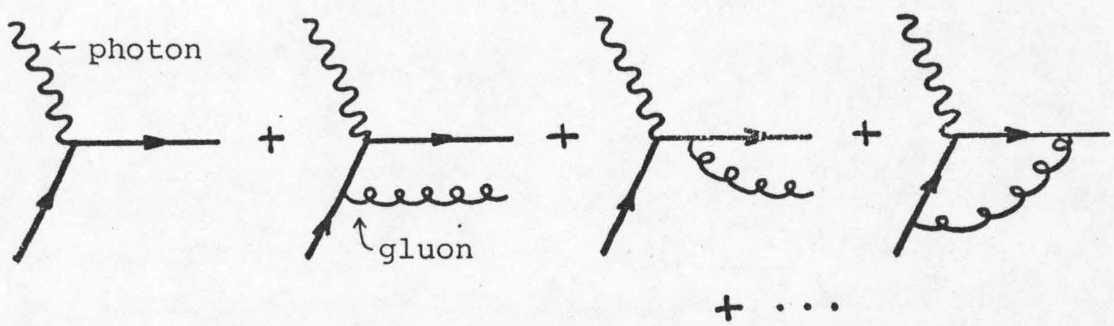


fig. 6

These depend logarithmically on Q^2. There is a certain convention dependence on what is to be called a bare or a dressed quark. If at some Q_0^2, $\nu W_2^{\text{quark}}(\frac{x}{z}, Q_0^2)$ is a δ-function, then for generic Q^2

$$\nu W_2^{\text{quark}}(\frac{x}{z}, Q^2) \sim \delta(\frac{x}{z} - 1) + g^2\, a(\frac{x}{z}) \log \frac{Q^2}{Q_0^2} + \ldots$$

For Q^2 very different from Q_0^2, the gluon radiative corrections can be large. There is a well-defined procedure of exponentiating the logs (via the renormalization group). The result of this exponentiation is that the natural variable for measuring scaling violations is dog Q^2, where

$$\text{dog } Q^2 \equiv \log \frac{\log \frac{Q^2}{\Lambda^2}}{\log \frac{Q_0^2}{\Lambda^2}}$$

and Λ is determined experimentally to be about 0.5 GeV.[3]

To emphasize the double log, I have reploted the Q^2-ν plot for various incident energies in fig. 7.

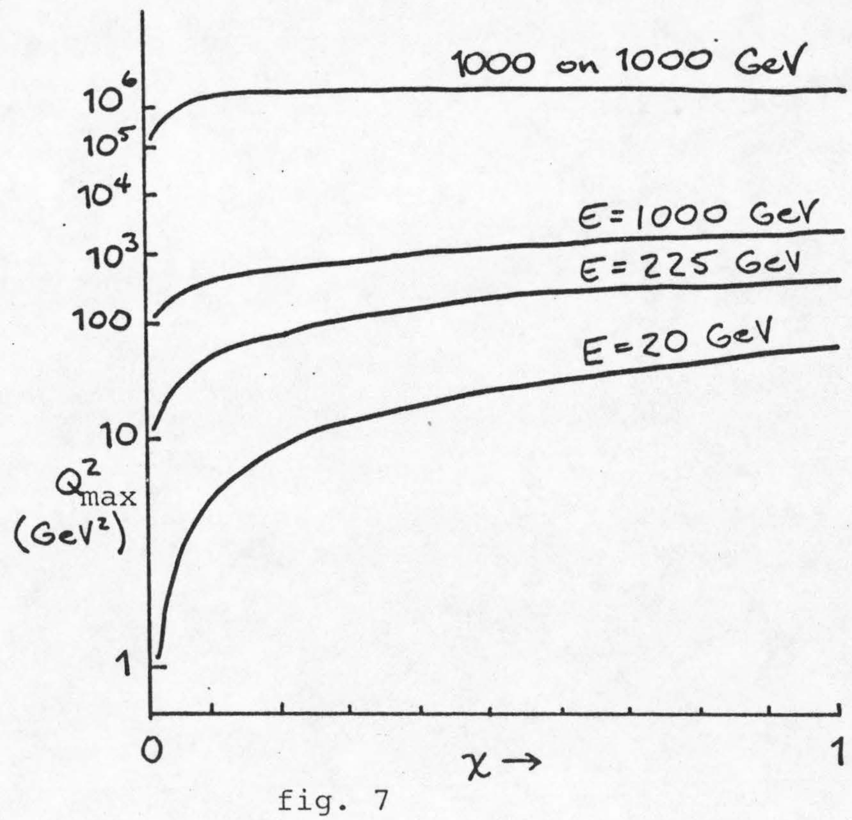


fig. 7

Note that, while Q^2/Q_0^2, $\log Q^2/Q_0^2$, and dog Q^2 all have unit slope at $Q^2 = Q_0^2$, dog Q^2 really slows down.

The predictions for νW_2 for the proton, based on the QCD analysis of SLAC electroproduction are shown in fig. 8

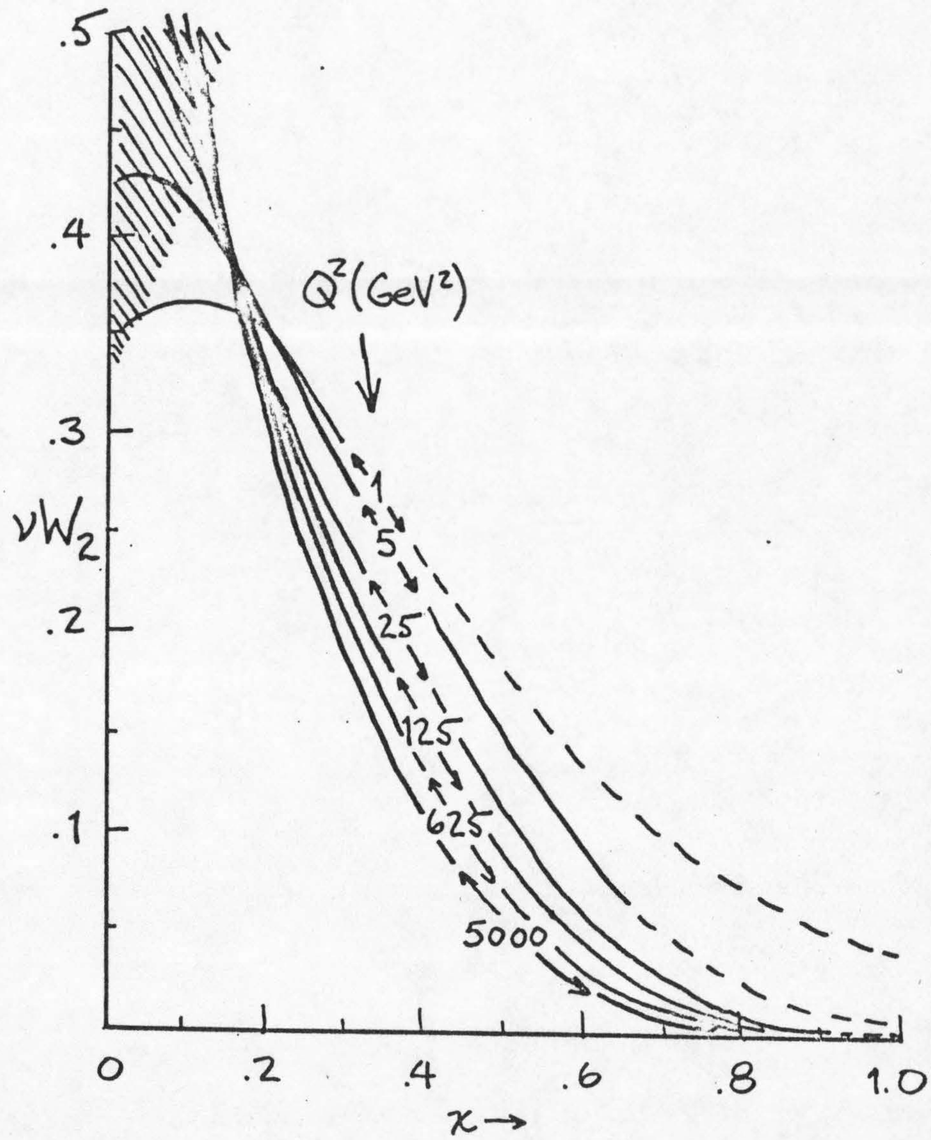


fig. 8

for different Q^2's. In the dotted regions one must smear over resonances. At higher and higher energies it becomes increasingly difficult to distinguish the predictions of QCD from exact scaling. This is because of the dog Q^2 dependence. Within a given experiment it will be very hard to see the scaling violations. What is needed is a good determination of the absolute normalization so that comparison can be made with lower energy experiments.

A comment on scaling variables: according to the field theory, the logarithmic radiative corrections should be measured using a variable[5]

$$\xi \approx \frac{x}{1 + x^2 \frac{m^2}{Q^2}} .$$

But this is clearly irrelevant for $Q^2 >> m^2$, the proton mass, and for $x^2 << 1$.

The small x region is shaded because scaling violations for $x \lesssim 0.2$ involve graphs of the form given in fig. 9,

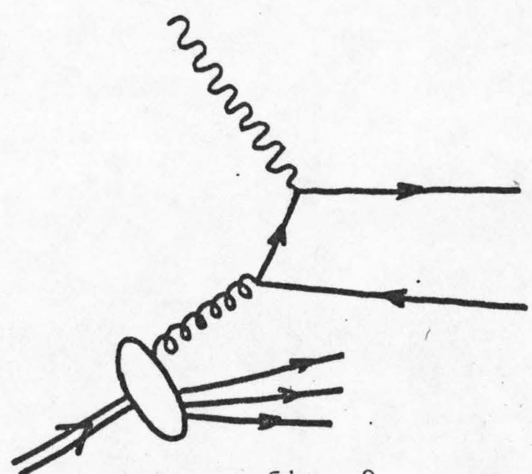


fig. 9

i.e. we need to know the distribution of gluons in the proton. While we do not yet know that distribution, we do know its integral, i.e. the total momentum carried by gluons[6] Hence we can predict $\int_0^1 dx\ \nu W_2$, which depends on Q^2 as the quarks and gluons reshuffle their momenta. Assuming the glue distribution is small above $x = 0.25$, we can also predict $\int_0^{1/4} dx\ \nu W_2$. Note the devastating effect of dog Q^2 in fig. 10.

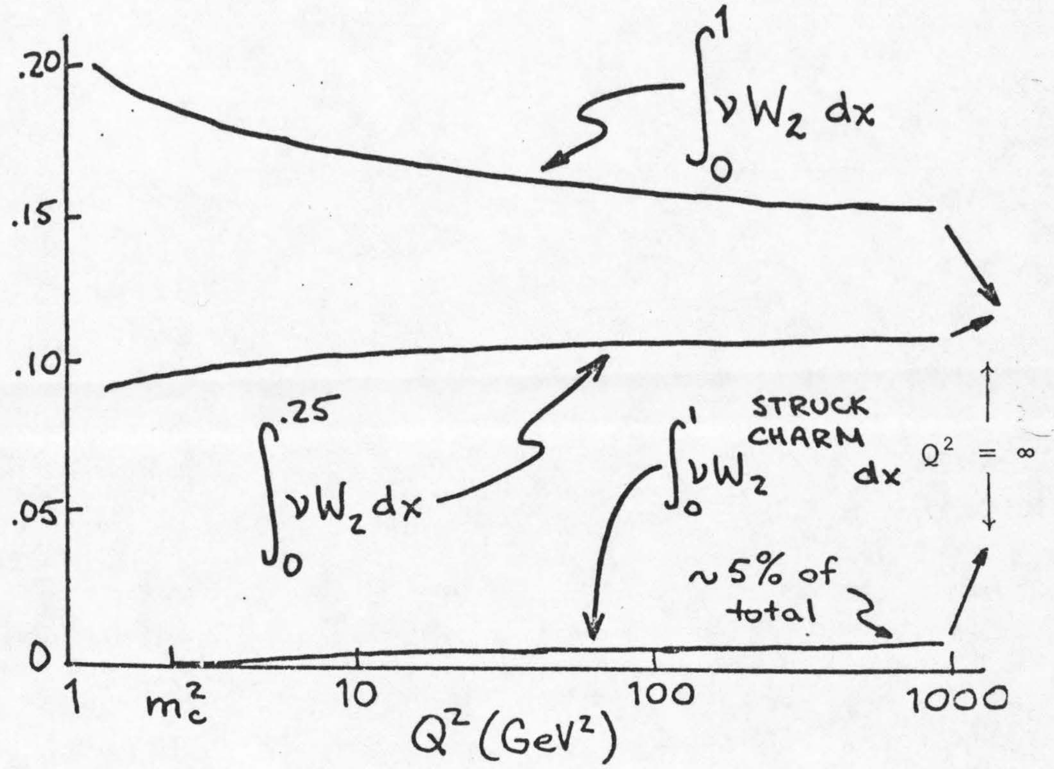


fig. 10

3. Other Tests of QCD

There are certain predictions of QCD which differ drasticly from the naive, non-interacting parton model. These have to do with transverse momenta, and the difference grows with energy. For "non-interacting" partons, (i.e. QCD with $g^2 \to 0$), transverse momenta are limited to a few hundred MeV (the inverse hadron size), independent of the energy of collision. In QCD, the same graphs which give scaling violations, e.g. figs. 6 and 9 give transverse momenta roughly equal to g times the energy of collision. At high energies, these large transverse momenta will be unmistakable.

Let me describe where these large momenta will show up: in σ_L/σ_T, in the broadening of the hadron jet in $\mu + p$ transverse to $\vec{q}$ in the lab, in two jet events in $\mu + p$, in a similar broadening and proliferation of jets in $e^+ + e^-$, in large transverse momenta ($\sim$ 20 GeV) of produced W bosons in $p + p(\bar{p}) \to W + X$ at 400 GeV (center of mass), and similarly in massive μ pair production in hadron-hadron collisions.

The predictions for σ_L/σ_T are theoretically clean[7] but experimentally hopeless,[4] I will, however, use them to describe the physics that underlies all these situations.

For a massless, free fermion, σ_L is zero. This follows from helicity conservation. In the naive parton model, σ_L for the proton can be non-zero if the quarks have some momentum transverse to the proton-photon axis. In such a case what is longitudinal with respect to the quark is no longer longitudinal with respect to the proton, in particular, fig. 11 gives[1]

$$\frac{\sigma_L}{\sigma_T} = \frac{4\langle \vec{p}_\perp^{\,2}(x)\rangle}{Q^2}$$

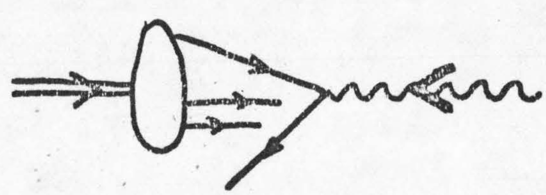


fig. 11

In QCD, fig. 12

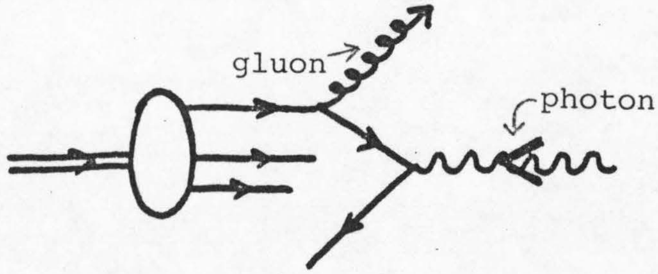


fig. 12

gives a scaling contribution to σ_L. The exact formulas are complicated (as functions of x) and depend on σ_T, but a crude interpolating formula is

$$\frac{\sigma_L}{\sigma_T} \approx \frac{1 - x}{2 \log Q^2/\Lambda^2} \approx \frac{1 - x}{2 \log 4Q^2} \, .$$

Hence gluon bremsstrahlung provides the quarks with a $\vec{p}_\perp^{\,2}$ of approximately

$$\langle \vec{p}_\perp^{\,2} \rangle \approx \frac{(1 - x)\, Q^2}{8 \log 4\, Q^2} \, .$$

So at high Q^2, the $\vec{p}_\perp^{\,2}$ is not set by the inverse hadron size but by the inverse size of the dressed quark that is resolved by the photon wave length. In QCD, quarks are made of quarks and gluons in the same way that in QED electrons are made of Dirac electrons and photons.

At $Q^2 \sim 1000$ GeV^2, the QCD prediction is enormous compared to the naive parton prediction, but it is experimentally indistinguishable from zero. See table 1.

Table 1

Q^2	$\sigma_L/\sigma_T(x = 0.5, Q^2)$
5	0.08
50	.05
500	.03

We can see the large parton transverse momenta directly in $\mu + p \to \mu + \pi + X$. The photon direction is determined by the muons alone. The mean π momentum transverse to that direction will be of order of the above-mentioned quark $p_\perp$. (More precisely it will be something like $z^2 \; \vec{p}^{\,2}_{\perp\text{parton}}$ convoluted with $D(z, Q^2, \vec{p}_\perp^{\,2})$, the probability that a parton will produce a π with a fraction z of its momentum).

Bremsstrahlung will give similarly large transverse momenta in Drell-Yan processes like massive μ-pair or W boson production.[8] See fig. 13.

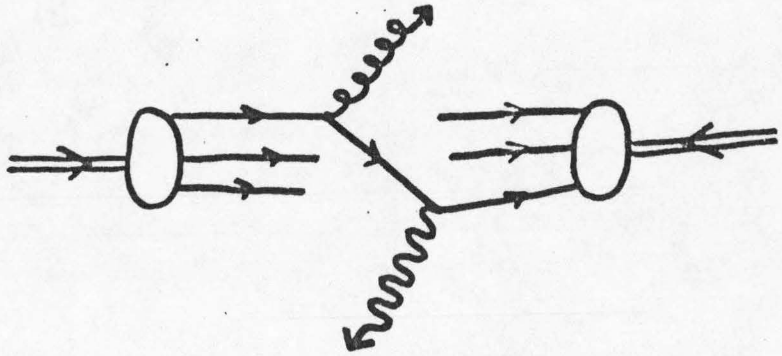


fig. 13

In $e^+ - e^-$ annihilation, parton processes such as in fig. 14

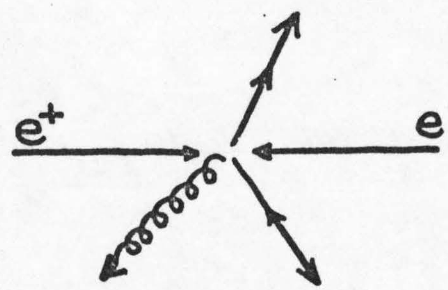


fig. 14

will similarly broaden the hadron jets and lead ultimately to three jet final states in roughly 10% of the events.

4. Weak Scattering of Muons

The relative size of $\mu + p \to \nu + X$ is given by the ratio of boson propagators. See table 2.

Table 2

Q^2	$\left(\frac{Q^2}{Q^2 + M_W^2}\right)^2$
5	10^{-6}
50	10^{-4}
500	10^{-2}
5000	0.20

With a 1000 GeV muon beam, there is little chance of seeing any really exotic weak process. The neutral current interference term is characterized by $Q^2/(Q^2+M_Z^2)$, which should be discernable with the proposed muon beams.

One exciting possibility for muon beams is to look for right-handed weak interactions initiated by right-handed muons. A beam of definite longitudinal polarization can be made without drastic loss of intensity. The point is, of course, that such interactions cannot be initiated with neutrino beams, yet they may in fact exist at typical weak strength for right-handed muons.

5. Multiple Muons and New Quarks

It is often asked whether there is any relation between the multiple muons in neutrino scattering and the multiple muons in muon scattering. The answer is probably complicated, so let me address it in some detail.

In neutrino scattering, the dominant sources of extra, prompt muons are the weak decays of new particles. These new particles can be leptons or hadrons. A single event could contain either, both, or several of each. Fig. 15 shows an event containing two

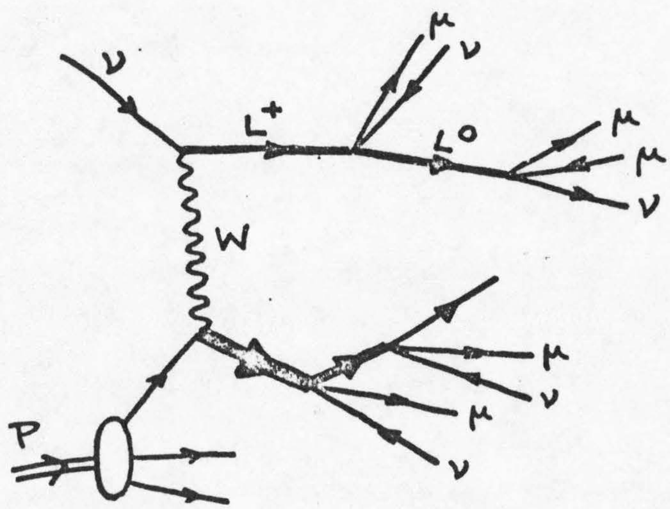


fig. 15

of each. A muon-induced analog of fig. 15 is shown in fig. 16.

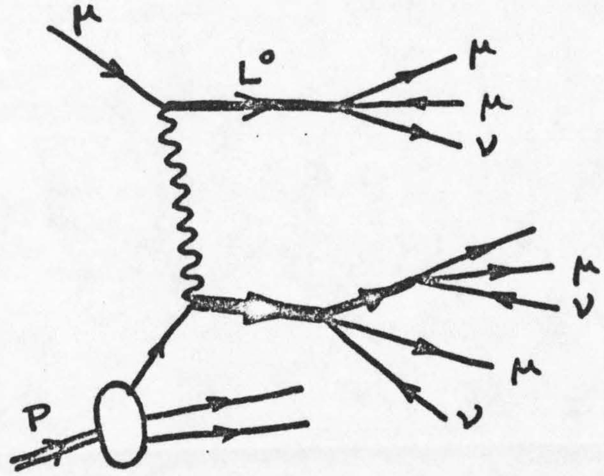


fig. 16

However, as a rare, weak event it is an unlikely competitor to associated production of new hadrons in electromagnetic scattering.

Such associated production can be devided into two classes: either 1) produced in the final state interaction or 2) summoned out of the "sea", i.e. the struck quark is heavy.

There is no detailed theory of associated production in the final state interaction, but it can be argued to be exponentially small (with the quark mass). We can imagine the production as the breaking of the color flux strings that connect the outgoing partons. The string will have an amplitude per unit length to break by producing a quark pair. That amplitude can depend only on the produced quark mass. Prodcution of heavy quarks will rely on a fluctuation in the energy density along the string so that the energy required to make the massive quarks is localized. In quantum mechanics, such fluctuations drop exponentially with the energy concentration required. Hence the production amplitude is likely proportional to e^{-m_i/m_0}, where m_i is the ith quark mass and m_0 is characteristic of the color interactions, say a few hundred MeV. (The same conclusion follows from a thermodynamic argument, where m_0 is the Hagedorn temperature). This suggests that the probability of charmed meson production relative to pion production is suppressed by a factor of 20,000 or much more (subject to the vagueries of the magnitudes of light quark masses).

If the new quark is the struck quark (fig. 17), then QCD can

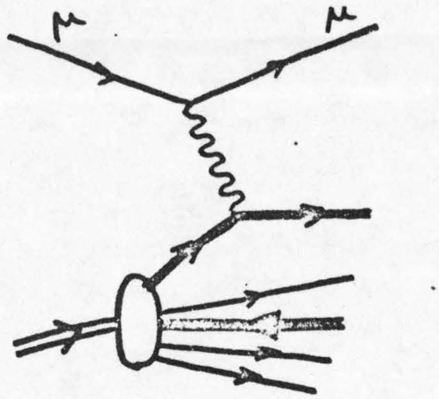


fig. 17

be used to predict the production cross section,[5,6,9] The result is much larger than the above string breaking estimate. The prediction includes processes of the types in fig. 18.

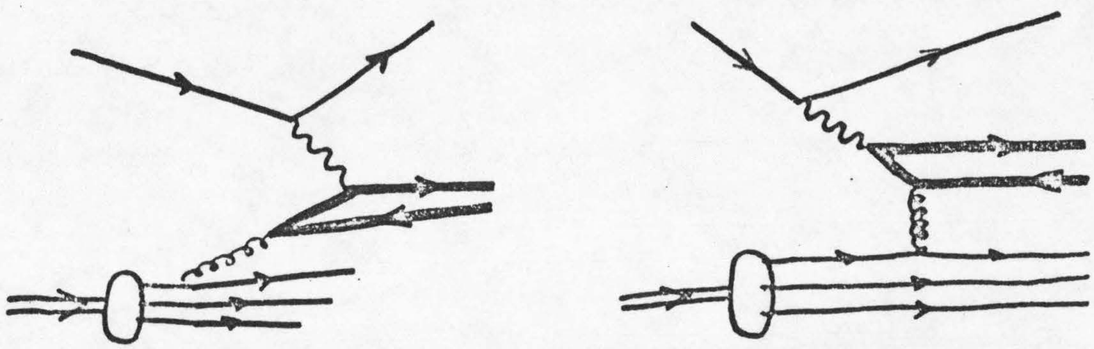


fig. 18

Since it does involve the gluon distribution, the most reliable prediction is for the integral over all x as a function of Q^2. See fig. 10. The x dependence is predicted to have yet one more power of $(1 - x)$, peaking at small x, than the gluon distribution. The production of quarks yet heavier than charm will be negligible until Q^2 exceeds the heavier mass-squared. The production will then increase with Q^2 much as charm production increases above m_c^2.

Extra muons will come from roughly 20% of the charm decays. At $Q^2 \gtrsim 10\ GeV^2$, charm is produced in roughly 2% of the muon scatterings. Hence dimuons are expected at a rate of about 4×10^{-3} and trimuons at about 8×10^{-4} per single muon event. The rates will double at $Q^2 \sim 500\ GeV^2$. (These compare very favorably with preliminary reports of di-and tri-muons.[10])

In hadron-hadron collisions, most of the quark momentum transfers are small. Therefore the string breaking picture is the dominant mechanism for charm production, and it is excruciatingly small. Clearly the place to look for charm is at high p_T, i.e. events involving large momentum transfers. There is still a big suppression factor because the charmed quarks in the sea reside at very small x.

6. Conclusion

Barring spectacular surprises, probably the most interesting feature of high energy muon beams will be the study of how an impulsed quark turns into hadrons, i.e. the nature of the final state. Unique to muon scattering is the fact that we can essentially measure the outgoing quark configuration immediately after the impulse, via the muon invariants, Q^2 and ν. So the jet axis is precisely determined by the muon. What are the detailed features of the jet? QCD makes some specific predictions.

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